UC UNIVERSIDAD DE CANTABRIA

CANTABRIA CAMPUS INTERNACIONAL

CIEM Centro Internacional de Encuentros Matemáticos

EXCMO. AYUNTAMIENTO DE CASTRO URDIALES

Las Rocas Playa Hotel CASTRO URDIALES

## Book of abstracts for the workshop:

# Modelling of nonlinear dispersive waves: Mathematical theory and numerical approximation.

**Dates**: May 27-29, 2019.

### Summary

Water wave propagation phenomena still attract the interest of researchers from many areas and with various objectives, including better experimental, physical and mathematical understanding of their dynamics, especially in near-shore zones, harnessing their power, improvement of the accuracy of wave field observations, risk assessment and development of emergency plans to prevent damage due to extreme phenomena like storm waves and tsunamis, etc. All objectives depend on a better mathematical description of the waves and their properties.

The mathematical modelling of wave propagation, either on the surface of a fluid or on the interface of immiscible fluid layers of different densities, is based in the simplest case on the Euler equations for irrotational waves in the relevant physical setting. In many applications, the full Euler equations are too complex to allow a detailed study of the dynamics of the problem, and the derivation of simpler, approximate partial differential equation (pde) models in various scaling regimes, is in order. The validity of these models as approximations of the Euler equations, their rigorous mathematical analysis, the study of their special solutions such as solitary waves, the construction and error analysis of efficient numerical methods for the simulation of their solutions, and the comparison of numerical predictions with experimental results are still challenging topics of intensive research.

The purpose of this workshop is to present modern aspects of the validity, pde theory and numerical analysis of such models of surface or internal water waves, that are usually represented by systems of nonlinear, dispersive wave equations and related hyperbolic equations. Internationally known researchers, mathematicians, physicists and engineers will be invited to attend, interact, and present their latest work in the area. The workshop will also have a strong training component as several young researchers (PhD candidates) working in the general area of nonlinear waves, will be invited to present their results and problems.

# Large amplitude mode-2 internal solitary waves in three-layer flows.

Ricardo Barros

Loughborough University, Loughborough LE11 3TU UK

We consider a strongly nonlinear long wave model for large amplitude internal waves in a three-layer flow bounded above and below by rigid boundaries. The model extends the two-layer Miyata-Choi-Camassa (MCC) model (Miyata 1988; Choi & Camassa 1999) and is able to describe the propagation of long internal waves of both the first and second baroclinic modes. Solitary-wave solutions of the model are shown to be governed by a Hamiltonian system with two degrees of freedom. Emphasis is given to the solitary waves of the second baroclinic mode (mode-2) and their strongly nonlinear characteristics that fail to be captured by weakly nonlinear models. In asymptotic limits relevant to oceanic applications and previous laboratory experiments, it is shown that, after choosing relevant physical parameters, large amplitude mode-2 waves with single-hump profiles can be described by the solitary wave solutions of the MCC model, originally developed for mode-1 waves in a two-layer system. As a result of the richness of the dynamical system with two degrees of freedom, in the case when the density stratification is weak and the density transition layer is thin, new classes of mode-2 solutions, characterized by multi-humped wave profiles of large amplitude, are also found. In contrast with the classical solitary-wave solutions described by the MCC equation, such multi-humped solutions cannot exist for a continuum set of wave speeds for a given layer configuration. Our analytical predictions based on asymptotic theory are then corroborated by a numerical study of the full dynamical system.

**Diffusive-dispersive travelling wave solutions**

Daria Bolbot

King Abdullah University of Science and Technology (KAUST)

It is commonly known fact that hyperbolic scalar conservation equations as well as such systems specifically with convex (concave) fluxes admit shock solutions. But to be able to consider mathematical systems of PDEs as a model of physical phenomena dissipative along with dispersive (at least weakly) terms should be considered. My research is related to analyzing the travelling wave solutions of hyperbolic systems regularized by weakly dispersive and weakly viscous terms. The issue in question is the convergence of travelling wave solutions to a an entropy admissible solutions of conservative systems. More specifically what is the threshold relation between diffusion and dispersive terms should take place for the solution to still be convergent. Both analytical and numerical approaches (such as Spectral methods and some Riemann solver techniques) are used in the process..

# Systems of Equations for Internal Waves

Jerry L. Bona

University of Illinois at Chicago

Large scale internal waves are frequently present in certain parts of our oceans. The lecture will begin with a precis of this phenomenon, followed by some recently derived models for internal wave propagation. The discussion will then turn to mathematical analysis concerning these coupled systems of partial differential equations which feature non-local operators. The discussion will conclude with a set of numerical simulations illustrating some of the spatial dynamics of solutions and applications of the model to some interesting oceanographic phenomena.

**How to avoid order reduction when Lawson methods integrate nonlinear initial boundary value problems**

Begoña Cano

Applied Mathematics Department, University of Valladolid, Spain

Lawson methods are a special kind of exponential methods. The latter have the advantage of being able to be explicit and stable for stiff problems at the same time. Therefore, they seem quite suitable for the integration of partial differential problems. However, it is well known that Lawson methods suffer from a severe order reduction when integrating initial boundary value problems where the solutions are not periodic in space or do not satisfy enough conditions of annihilation on the boundary. We suggest a technique to avoid that order reduction. This is very useful because, given any Runge-Kutta method of any classical order, a Lawson method can be constructed associated to it for which the order is conserved.

**Modeling tsunamis generated by submerged landslides in shallow-flows**

M.J. Castro[1], C. Escalantey[2], T. Morales de Luna[3], J. Macías[1], E.D. Fernández-Nieto[4]

[1] Dpto. AM,EIO y Matemática Aplicada. University of Málaga, Spain

[2] Dpto. Matemática Aplicada. University of Málaga, Spain

[3] Dpto. Matemática. University of Córdoba, Spain

[4] Dpto. Matemática Aplicada. University of Sevilla, Spain

In this talk we present a family of models for the simulation of landslide generated tsunamis. All of them fit in the framework of shallow-flows. Here, the flow is supposed to be modeled by the multilayer shallow-water system with hydrostatic or non-hydrostatic pressure (see [3]). Multilayer shallow-water models allow us to recover the vertical profile of the velocities, that may be relevant at the early stages of the landslide-fluid interaction. Concerning the evolution of the landslide, either it is considered to be a rigid body and its motion it is supposed to be known, either it is supposed to be modeled by a Savage-Hutter type model ([2]). The resulting system its non-conservative and its discretized using a high-order path-conservative scheme ([1]). Finally, an exhaustive validation procedure has been carrying out by the comparison with laboratory experiments described in ([4]) and real events over real bathymetries.

**References**

[1] M. J. Castro, T. Morales de Luna, C. Parés. *Handbook of Numerical Methods for Hyperbolic Problems. Vol. 18, Applied and Modern Issues: Well-Balanced Schemes and Path-Conservative Numerical Methods*, pages 131-175. Handbook of Numerical Analysis. North Holland, 2017. ISBN: 9780444639103.

[2] E. D. Fernández-Nieto, F. Bouchut, D. Bresch, M. J. Castro, A. Mangeney. A new Savage-Hutter type models for submarine avalanches and generated tsunami. *J. Comput. Phys*., 227:7720-7754, 2008.

[3] E.D. Fernández-Nieto, M. Parisot, Y. Penel, and J. Sainte-Marie. Layer-averaged approximation of Euler equations for free surface flows with a non-hydrostatic pressure. *Communications in Math. Sciences*, 2018, 16(5): 1169{1202, 2018.

[4] G. Ma, F. Shi, and J.T. Kirby. Shock-capturing non-hydrostatic model for fully dispersive surface wave processes, *Ocean Modelling*, 43-44:22-35, 2012.

**Stability and instability of solitary-wave solutions to coupled systems of nonlinear dispersive equations**

Hongqiu Chen

University of Memphis

The classical Korteweg-de Vries equation (KdV for short) and its alternative, the regularized long-wave equation (RLW), also called the Benjamin, Bona, Mahony (BBM) equation, are mathematical models that approximately describe surface waves in shallow water in certain regimes .Both the KdV and BBM equations are globally well-posed in a wide range of Sobolev spaces and possess solitary-wave solutions. Furthermore, solitary-wave solutions are all stable.

Systems of coupled KdV equations and coupled BBM equations appear in various applications. There is far less known about them and it appears that their theory is considerably more complicated. The focus of my presentation is existence, stability and instability of solitary-wave solutions for systems of these sorts.

## **Whitham--Boussinesq models: well-posedness and numerical simulation**

Evgueni Dinvay[1], Henrik Kalish[1], Achenef Temesgen[1], Denys Dutykh[2]

[1]Department of Mathematics, University of Bergen, Norway
[2] LAMA, UMR5127, CNRS Université Savoi Mont Blanc, France

We consider a class of dispersive systems modelling surface waves of an inviscid incompressible fluid. Being fully dispersive in the linear part,these models have nonlinear parts that are asymptotically equivalent to the standard Boussinesq nonlinearity. They can be regarded as bi-directional extensions of the so called Whitham equation. Their validity is supported by implementation of the weakly nonlinear approximation applied to the Hamiltonian formulation of the water wave problem

$$\eta_t = \frac{\delta H}{\delta \varphi}, \quad \varphi_t = -\frac{\delta H}{\delta \eta}$$

Changing only the total energy, H, while staying in the same framework of accuracy, one can arrive at different Whitham-Boussinesq systems.This accuracy is checked by comparing with numerical solutions of the Euler system given by the conformal mapping technique and with laboratory experiments.

Special attention is paid to the system

$$\eta_t = -v_x - i \tanh D(\eta v)$$

$$v_t = -i \tanh D\, \eta - i \tanh D\, v^2/2$$

where $D = -i\, \delta_x$ and tanh D is a Fourier multiplier operator in $L_2(R)$. Here $\eta$ denotes the surface elevation. Its dual variable v has the meaning of the surface fluid velocity.We give a proof of the local well-posedness of the initial value problem for the system in classical Sobolev spaces implementing dispersive estimates of Strichartz type with the fixed point argument. Conservation of the Hamiltonian allows us to extend globally well-posedness at least for small initial data.

The presentation is a summary of the papers [1,2,3].

**References**

[1] E. Dinvay, A. Tesfahun, Small data global well-posedness for a dispersive system of the Whitham--Boussinesq type, Preprint.

[2] E. Dinvay, On well-posedness of a dispersive system of the Whitham--Boussinesq type, Applied Mathematics Letters,Volume 88, February 2019, Pages 13-20.

[3] E. Dinvay, D. Dutykh, H. Kalisch, A comparative study of bi-directional Whitham systems, Applied Numerical Mathematics, to appear.

**Galerkin-finite element methods for the Camassa-Holm equation**

Vassilios A. Dougalis

Mathematics Dept., University of Athens, and IACM-FORTH, Greece

In this talk joint work with D.C.Antonopoulos and D.E. Mitsotakis will be presented on the Camassa-Holm (CH) equation, a nonlinear dispersive wave equation that models one-way propagation of long waves of moderately small amplitude. We discretize in space the periodic initial-value problem for CH (written in its original and in system form), using the standard Galerkin finite element method with smooth splines on a uniform mesh, and prove optimal-order L2-error estimates for the semidiscrete approximation.We also consider an initial-boundary-value problem on a finite interval for the system form of CH and analyze the convergence of its standard Galerkin semidiscretization. Using the fourth-order accurate, explicit, "classical" Runge-Kutta scheme for time-stepping, we construct a highly accurate, stable, fully discrete scheme that we employ in numerical experiments to approximate solutions of CH, mainly smooth travelling waves and nonsmooth solitons of the 'peakon' type.

**On solitary wave solutions of Boussinesq-Full dispersion systems for internal waves**

A. Durán

University of Valladolid, Spain

This talk is concerned with the generation and dynamics of solitary wave solutions of a two-layer system for the propagation of internal waves under the Boussinesq-Full dispersion (BFD) regime, proposed in [2]. Existence of solitary wave solutions, regularity and decay at infinity results for the Hamiltonian case were recently proved in [1] by using the concentration-compactness theory, [3]. The talk will present a numerical study with two parts. In the first one, the extension of these existence results is conjectured by studying the convergence of the Petviashvili's iteration, [4], to generate numerically the solitary wave profiles. The second part of the study is devoted to the dynamics of the solitary waves. A plethora of experiments aims at analyzing, by computational means, the stability under small perturbations, the collisions of solitary waves and the resolution property.

**References**

[1] J. Angulo-Pava, J.-C. Saut, Existence of solitary waves solutions for internal waves in two-layers systems, Quaterly of Applied Mathematics. 2019, to appear.

[2] J. L. Bona, D. Lannes, J.-C. Saut, Asymptotic models for internal waves, J. Math. Pures Appl., 89 (2008), 538-566.

[3] P.-L. Lions, The concentration-compactness principle in the calculus of variations. The locally compactness case, part 1, Ann. Inst. H. Poincaré, Anal. Non Linéare, 1 (1984), 109-145.

[4] V. I. Petviashvili, Equation of an extraordinary soliton, Sov. J. Plasma Phys. 2(1976), 257-258.

.

**Asymptotic models for free boundaries in incompressible flows**

Rafael Granero Belinchón

University of Cantabria, Spain

In this talk we will present some new asymptotic models for free boundary Euler and Darcy flows. Furthermore, we will also introduce a model of water waves with viscosity. We will also review some recent mathematical results concerning these nonlocal and nonlinear evolutionary equations.

# Finite Element Methods for a system of Dispersive Equations.

Ohannes Karakashian

University of Tennessee

The talk is concerned with the numerical approximation of periodic solutions of systems of Korteweg--de Vries type, coupled through their nonlinear terms. We construct, analyze and numerically validate two types of schemes that differ in their treatment of the third derivatives appearing in the system. One approach preserves a certain important invariant of the system, up to round-off error, while the other, somewhat more standard method introduces a measure of dissipation. For both methods, we prove convergence of a semi-discrete approximation and highlight differences in the basic assumptions required for each. Numerical experiments are also conducted with the aim of ascertaining the accuracy of the two schemes when integrations are made over long time intervals.

Authors: Jerry Bona, Hogqiu Chen, Ohannes Karakashian (speaker), Michael Wise.

**Galerkin finite element methods for Boussinesq-Peregrine type systems**

Theodoros Katsaounis

King Abdullah University of Science and Technology (KAUST)

We investigate the application of the standard Galerkin finite element method to Boussinesq-Peregrine type systems in 2D. We consider flat and variable bottom topographies and we study the convergence properties of the method. Various numerical experiments are presented and the compared with experimental data on flat and variable bathymetry. This is joint work with D. Mitsotakis.

**Galerkin methods for the Classical Boussinesq system over a variable bottom**

Grigorios Kounadis

Mathematics Dept., Univ.of Athens, and IACM-FORTH, Greece

In this talk, we consider the classical Boussinesq system for surface water waves in a finite channel with a variable bottom that can be either weakly nonlinear or fully non linear (Peregrine model). We discretize the problem using the standard Galerkin method in space and the classical, 4-stage, $4^{th}$- order explicit Runge-Kutta method for time stepping. We prove error estimates for the semidiscretization of a simple initial-boundary-value problem for the system, and present numerical experiments, in which the system is posed with approximate absorbing boundary conditions at the endpoints of the channel, in order to simulate the evolution of solitary waves over variable bottom topography. We compare our results with those of Peregrine 1967 and Madsen & Mei 1969 and with analogous results for the Serre system.

**Convergence of Petviashvili's method near periodic waves**

D. Pelinovsky

McMaster University

Petviashvili's method has been successfully used for approximating of solitary waves in nonlinear evolution equations. It was discovered empirically that the method may fail for approximating of periodic waves. We consider the case study of the fractional Korteweg-de Vries equation and explain divergence of Petviashvili's method from unstable eigenvalues of the generalized eigenvalue problem. We also show that a simple modification of the iterative method after the mean value shift results in the unconditional convergence of Petviashvili's method.

**Error estimates for spectral semidiscretizations of Boussinesq systems**

Leetha Saridaki

Mathematics Dept., University of Athens, and IACM-FORTH, Greece

We consider the periodic initial-value problem for the a-b-c-d system of Boussinesq equations , we discretize it in space by the Fourier- Galerkin spectral method and prove error estimates for various classes of the parameters a,b,c,d.

**On some internal waves systems**

Jean-Claude Saut

University Paris-Sud

We will review results on the Cauchy problem and solitary waves for systems modeling internal waves, including the system versions of the Benjamin-Ono and ILW equations.