Modelling of fluid propagation: mathematical theory and numerical approximation

Castro Urdiales, June 16-19, 2025

CIEM
Centro Internacional de Encuentros Matemáticos

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Abstracts

A geometrical Green-Naghdi type system for dispersive-like waves in prismatic channels

Monday
14:30

Sergey Gavrilyuk and Mario Ricchiuto

Aix-Marseille University

We consider 2D free surface gravity waves described by the Saint-Venant equations (shallow water equations) in prismatic channels with periodic bathymetric variations in the transverse direction. Averaged in the transverse direction, the corresponding 1D equations represent a fully nonlinear model describing dispersive effects strictly related to geometrical variations of the channel topography. The model is Galilean invariant and admits a variational formulation under natural assumptions about the channel geometry. It is endowed with an exact energy conservation law, and admits exact travelling wave solutions. The system is recast in two useful forms appropriate for its numerical approximations, whose properties are discussed. Numerical results allow to verify our implementation and validate our model against fully 2D nonlinear shallow water simulations.

Internal waves in viscous fluids

Monday
15:10

Rafael Granero-Belinchón

University of Cantabria, Spain

In this talk we will present some new results regarding the dynamics of the internal waves between to immiscible viscous fluids with different densities. This problem is also called in the literature the Stokes-transport problem. In particular, we will deal with the global well-posedness and stability questions.

Monday
16:20

Long nonlinear ring waves of small and moderate amplitude

Karima Khusnutdinova
Loughborough University, Loughborough LE11 3TU, UK

In this talk I will give an overview of the recent developments in the studies of long nonlinear ring waves, and the related developments for perturbed line solitons and hybrid waves consisting, to leading order, of a part of a ring wave and two tangent plane waves. For the waves propagating in a stratified fluid over a parallel shear flow there exists a linear modal decomposition (separation of variables) in the far-field set of Euler equations with the boundary conditions appropriate for oceanographic applications, more complicated than the known decomposition for plane waves [1]. The modal equations constitute a new spectral problem and lead to the need to construct a singular solution of a nonlinear first-order ODE responsible for the adjustment of the speed of the wave in different directions. Two different evolution equations can be obtained depending upon whether the coefficients are evaluated using the general, or singular, solution of this equation [2, 3]. The range of validity of these models is established with the help of direct numerical simulations within 2D Boussinesq-type systems for surface waves [4, 5]. Analytical results for perturbed line solitons are obtained using a non-trivial mapping to the KdV equation similar to that noted in [6], which retains dependence on the polar angle. Interesting results are obtained for the hybrid waves. The outward / inward propagating hybrid waves are shown to be stable / unstable to a localised perturbation, with a very interesting evolution scenario for the inward propagation. We confirm some predictions made by Ostrovsky and Shrira [7], and shed light on the developed stage of the instability. Finally, we show, in a case study, that the extended cKdV model provides a much more accurate description of the surface ring waves and allows one to extend the range of validity of the weakly-nonlinear modelling to the waves of moderate amplitude [4].

References:

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[4] N. Sidorovas, D. Tseluiko, W. Choi, K. Khusnutdinova, Nonlinear concentric water waves of moderate amplitude, Wave Motion 128 (2024) 103295.

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[6] R.S. Johnson, Ring waves on the surface of shear flows: a linear and nonlinear theory, J. Fluid Mech. 215 (1990) 145-160.

[7] L.A. Ostrovsky, V.I. Shrira, Instability and self-refraction of solitons, Sov. Phys. JETP 44 (1976) 738-743.

Recent results on extreme Stokes waves.

Monday
17:00

Eugeniy Lokharu
Lund University

In this talk we will discuss some of the recent progress on extreme Stokes waves with vorticity. We will prove the existence of extreme waves in the case of favorable vorticity and will touch upon some new results for flows with critical layers.

Instability of the peaked traveling wave in a local model for shallow water waves

Monday
17:40

Dmitry Pelinovsky
McMaster University

The traveling wave with the peaked profile arises in the limit of the family of traveling waves with the smooth profiles. We study the linear and nonlinear stability of the peaked traveling wave by using a local model for shallow water waves, which is related to the Hunter-Saxton equation. Within the linearized equations, we prove the spectral instability of the peaked traveling wave from the spectrum of the linearized operator in a Hilbert space, which completely covers the closed vertical strip with a specific half width. Within the nonlinear equations, we prove the nonlinear instability of the peaked traveling wave by showing that the gradient of perturbations grows at the wave peak. We also show that the spectral instability of the peaked traveling wave cannot be obtained in the limit along the family of the spectrally stable smooth traveling waves.

Weakly dispersive perturbations of nonlinear hyperbolic equations

Tuesday
9:00

Jean-Claude Saut
University of Paris-Saclay, France

We will present results describing the influence of a weakly dispersive perturbation on nonlinear hyperbolic equations or systems, in particular on the possible appearance of shock solutions.

Tuesday
9:40

Blood Rheology Modeling and Simulations

Adélia Sequeira
University of Lisbon, Portugal

Experimental studies over many years have shown that blood flow exhibits non-Newtonian characteristics such as, shear-thinning, thixotropy, viscoelasticity and a yield stress. The complex rheology of blood is influenced by numerous factors, including plasma viscosity, hematocrit and in particular, the ability of erythrocytes to form aggregates when at rest or at low shear rates and to deform at high shear rates, storing and releasing energy. Hemodynamic analysis of blood flow in vascular beds and prosthetic devices requires the rheological behavior of blood to be characterized by phenomenological constitutive equations relating the stress to the rate of deformation and flow. In this talk we present a short overview of some macroscopic constitutive models that can mathematically characterize the rheology of blood and describe their known phenomenological properties. Some test cases formulated in idealized and anatomically realistic vessels will be considered to investigate the impact of the most significant non-Newtonian characteristics of blood on its flow behavior, based on numerical simulations of different blood constitutive equations under given sets of physiological flow conditions.

Tuesday
10:50

The 2D nonlinear shallow water equations with a partially immersed obstacle

David Lannes
University of Bordeaux, France

In this talk I will present two works with T. Iguchi on the well-posedness of the 2D nonlinear shallow water (NSW) equations in the presence of a partially immersed obstacle. When the object is fixed and the side walls are vertical, we show how to reduce the problem to a mixed (initial and boundary value) problem for the NSW equations in an exterior domain. The boundary condition are nonlocal in time and space and are not covered by classical theories, and we are in a situation where the eigenvalues of the boundary matrix change sign and the construction of a Kreiss symmetrizer remains an open problem; to bypass this difficulty, we introduce the notion of weakly dissipative boundary conditions. When the sidewalls of the object are not vertical, one has in addition to handle the dynamics of the contact line; this requires new hidden regularity results that allow the control of the trace of the surface parametrization at the boundary.

Boundary value problems arising in magnetohydrodynamics

Tuesday
11:30

Diego Alonso-Orán
University of La Laguna, Spain

Magnetohydrodynamics plays a crucial role in understanding the behavior of plasmas, electromagnetic fields, and fluid dynamics, providing a fundamental framework for studying phenomena in astrophysics, fusion energy, and space exploration. In this talk, we will present a short survey about the well-posedness of boundary value problems for steady ideal magnetic fluids. More precisely, we will first focus on the magneto-hydrostatic equations (in two and three dimensions) and conclude with some ongoing works related to the steady magneto-hydrodnamic equations. Joint works with D. Sánchez Simón del Pino, Juan J.L. Velázquez and E. Wahlén.

Linearisable Abel equations and the Gurevich-Pitaevskii problem

Tuesday
12:10

Evgeny Ferapontov
Loughborough University, UK

Applying symmetry reduction to a class of SL(2, R)-invariant third-order ODEs, we obtain Abel equations whose general solution can be parametrised by hypergeometric functions. Particular case of this construction provides a general parametric solution to the Kudashev equation, an ODE arising in the Gurevich-Pitaevskii problem, thus giving the first term of a large time asymptotic expansion of its solution in the oscillatory (Whitham) zone.

Based on Joint work with Stanislav Opanasenko:

S. Opanasenko, E.V. Ferapontov, Linearisable Abel equations and the Gurevich-Pitaevskii problem, Stud. Appl. Math. 150 (2023), no. 3, 607-628.

On the precise cusped behaviour of extreme solutions to Whitham-type equations

Tuesday
14:30

Mats Ehrnström
Trondheim University

We prove exact leading-order asymptotic behaviour at the origin for nontrivial solutions of two families of nonlocal equations. The equations investigated include those satisfied by the cusped highest steady waves for both the uni- and bidirectional Whitham equations. The problem is in a way analogous to capturing the 120◦ interior angle at the crests of classical Stokes' waves of greatest height, but as we shall see, requires several additional steps. Our results partially settle conjectures for extreme waves posed in a series of recent papers. The methods may be generalised to solutions of other nonlocal equations, and can moreover be used to determine asymptotic behaviour of their derivatives to any order.

This is joint work with O Mæhlen (Paris–Saclay) and K Varholm (Pittsburgh).

Tuesday
15:10

High-Order Accurate Discontinuous Galerkin Method for Light Propagation in Curved Biological Domains

Adérito Araújo, Sílvia Barbeiro, Milene Santos
CMUC, Department of Mathematics, University of Coimbra, Portugal

To model the incidence and reflection of light in the cornea, we employ Maxwell's equations, which govern the propagation of electromagnetic waves. In this talk, we focus on the time-harmonic form of Maxwell's equations, which leads to the Helmholtz equation [1]. Discontinuous Galerkin (DG) methods are widely recognized for their effectiveness in solving partial differential equations on complex geometries. However, they typically require the computational mesh to align with the geometry, which poses challenges when aiming to maintain optimal convergence in curved domains and across curved interfaces - a well-known issue, particularly relevant in modeling biological tissues. We propose a numerical approach based on nodal discontinuous Galerkin methods, combined with a strategy tailored to handle the naturally occurring curved geometries in our application. This approach incorporates the DG-ROD (Reconstruction for Off-site Data) method [2], which applies a polynomial reconstruction of boundary conditions on the computational domain. Crucially, this technique does not depend on curved meshes and preserves both the stability and optimal convergence properties of the DG method.

References:

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Tuesday
16:20

Homogenized models for water waves and gas dynamics with in a periodically-varying background

David Ketcheson
KAUST

Nonlinear hyperbolic PDEs in one dimension generically develop shock discontinuities in finite time, and their behavior for long times is characterized by entropy decay and irreversibility. However, in the presence of a spatially-periodic variation of the background state, solutions of these problems can instead exhibit solitary wave formation and energy conservation, similar to solutions of dispersive wave equations. I will consider two examples of this: water waves over a periodic bottom, and gas dynamics with a periodically-varying entropy. In both cases, I will show how to apply multiscale asymptotic analysis to derive an effective constant-coefficient dispersive PDE whose solution closely approximates that of the variable-coefficient hyperbolic problem.

Hyperbolic problems set in moving domains

Wednesday
9:00

Ana Carpio
Universidad Complutense de Madrid, Spain

In contrast with the huge amount of results available for the theoretical and numerical study of initial value problems for partial differential equations set in spatial regions whose boundaries do not change with time, little is known when boundaries move. The few available results are typically constrained to parabolic problems. We consider here hyperbolic problems, such as wave and transport equations. First, we will show how to construct solutions of initial value problems for linear wave equations in domains whose boundaries deform with time following a smooth vector field. Next, we will address more complex nonlinear transport problems arising a cellular studies which can be solved numerically by physically inspired changes of variables to produce travelling wave solutions. Finally, we will show how to handle different biologically inspired boundary dynamics by means of characteristic curves and discuss the limitations of the presented approaches.

References:

[1] B. Birnir, A. Carpio, G. Duro, Driving biofilms to finite time extinction by antibiotic cocktails, preprint, 2025.

[2] A. Carpio, G. Duro, On the solution of boundary value problems set in domains with moving boundaries, Mathematical Methods in Applied Sciences, to appear 2025

[3] R. Gonz´alez-Albaladejo, F. Ziebert, A. Carpio, Two-fluid variable length model for cell crawling, in C. Par´es et al. (eds.), Hyperbolic Problems: Theory, Numerics, Applications. Volume II, SEMA SIMAI Springer Series 35, Springer Nature Switzerland AG 2024.

[4] A. Carpio, G. Duro, Well posedness of fluid-solid mixture models for biofilm spread, Applied Mathematical Modelling 124, 64-85, 2023

On the ε-dependent Cahn-Hilliard/Allen-Cahn equation.

Wednesday
9:40

Dimitra Antonopoulou
National and Kapodistrian University of Athens

We consider the Cahn-Hilliard/Allen-Cahn equation, with weights $\delta(\varepsilon) > 0$ and $\mu(\varepsilon) >= 0$ on the Cahn-Hilliard and Allen-Cahn operators, respectively; $\varepsilon > 0$ is a small positive parameter that stands as the order of the width of the transitional layers during the phase separation of a binary alloy. In the context of the continuous problem, we discuss first the spectral condition that identifies the two main approximate manifold profiles for the solution which is either close to Allen-Cahn when μ dominates δ, or, close to the Cahn-Hilliard solution when δ dominates μ. The equation is then numerically approximated by an algebraically stable Runge-Kutta-SAV method for which optimal error estimates are proven. As the numerical simulations show, the phase-transitions seem to change significantly depending on the selection of the weights δ, μ and the parameter ε.

[This talk presents results in collaboration with Dongfang Li, Georgia Karali, and Kostantinos Tzirakis.]

Wednesday
10:50

Structure preserving methods for Schrodinger type of equations.

Theodoros Katsaounis
University of Crete, Greece

I will present a class of structure preserving methods for nonlinear Schrodinger equation modelling the generation of rogue waves and the Schrodinger-Poisson system with applications in cosmology. Joint work with I. Kyza and A. Athanassoulis.

Wednesday
14:30

Well-posedness of the stratified Euler equations

Théo Fradin
University of Bordeaux, France

The dynamics of the ocean can be described by the incompressible Euler equations, completed by a transport equation on the density. For large scale dynamics, the ocean is assumed stratified, meaning that it is layered by sheets of constant density, namely isopycnals. The aim of this talk is to present the study of perturbations around shear flows, which are natural equilibria of this system.

Wednesday
14:50

Well-posedness of a semilinear parabolic equation arising from the modelling of atmospheric flows

Luigi Roberti
University of Vienna

Recently, Constantin and Johnson derived a model describing 'morning glory clouds', a spectacular atmospheric phenomenon whereby elongated tubular clouds travel perpendicularly to the cloud line, in an essentially two-dimensional motion. Mathematically, the problem can be reduced to studying a semilinear parabolic equation with nonlocal nonlinearity. The goal of this talk is to illustrate some recent results concerning the well-posedness of this problem, including local strong well-posedness and existence of global weak solutions as well as global strong well-posedness for small initial data.

Solitary axisymmetric capillary water waves.

Wednesday
15:10

Stefano Böhmer, Dan Hill and Dag Nilsson
Lund University

We consider steady axisymmetric water waves subject to surface tension, where we study the free-boundary problem for domains close to an infinite cylinder. In the case of linear vorticity in radial direction and no swirl, we are able to prove existence of small solitary solutions of KdV-type. They bifurcate from laminar flows in a flat cylinder and the presence of vorticity is required for their existence. The proof relies on a spatial dynamics approach allowing for a center-manifold reduction, which reduces the problem to a finite-dimensional dynamical system. Homoclinic solutions of this system, which correspond to solitary wave solutions on the cylinder, are found using dynamical systems methods.

Asymmetric Travelling Capillary-Gravity Waves

Wednesday
15:30

Douglas Svensson Seth
NTNU Norwegian University of Science and Technology

Periodic travelling waves that solve the capillary-gravity Whitham equation have been fully characterised in the case of small and even waves. This characterisation is complemented by the work presented in this talk dealing with small asymmetric periodic travelling waves. Such asymmetric waves are far more scarce than the even ones and can only be constructed in certain cases for weak surface tension. The method also generalises in a straightforward way to a class of similar equations for which we either can prove the existence of or non-existence of asymmetric solutions. However, the proof relies on some technical calculations that are different for each equation. We discuss how this can be done for the Babenko equation, which is equivalent to the full water wave problem, to determine the existence of small amplitude Capillary-Gravity Waves.

Precipitation in nonlinear mountain waves with temperature-dependent enthalpy

Wednesday
15:50

Jordan McCarney
University College Cork

Mountains play a crucial role in shaping regional climates. For example, precipitation may occur when a moist air mass is forced over a mountain range, where the associated cooling as it rises results in the air becoming saturated. In this talk, we will consider an exact solution to the nonlinear governing equations for mountain waves in the Lagrangian framework. The waves propagate in a moist atmosphere, and we incorporate a temperature-dependence in the enthalpy of condensation of water vapour. We show that if a streamline in the atmospheric flow at a given temperature is saturated, then the explicit specification of the dynamics of the flow is sufficient to deduce the saturation temperature for any other streamline in the flow, and we find an expression for the corresponding vapour pressure at saturation for the flow also. We further deduce a restriction on the domain for which precipitation is admissible, enforced by the second-law of thermodynamics.

This is based on joint work with Tony Lyons

Wednesday
16:40

Adaptive Numerical Discretization of Dispersive Water Wave Models via Hyperbolic Relaxation

Carlos Muñoz-Moncayo, David Ketcheson

King Abdullah University of Science and Technology (KAUST)

The shallow water equations are a first-order hyperbolic system of PDEs for free-surface water wave modeling. Despite being one of the most widely used models for the study of geophysical flows in hazard mitigation, the shallow water equations do not account for dispersion. This could result in an imprecise estimation of coastal run-up when modeling phenomena that involve shorter-wavelength perturbations. On the other hand, dispersive water wave models do not develop shocks, which is a desirable property to mimic physical wave-breaking. Moreover, their formulation often involves systems of PDEs with high-order and mixed space-time derivatives or differential constraints with no time evolution. This can complicate the development of large-scale and efficient numerical discretizations due to the need to invert differential operators implicitly. Hyperbolic approximations of dispersive water wave models have been put forward in recent years, raising the question of whether such structure can be exploited for the development of explicit shock-capturing numerical schemes. In this work, we are concerned with the development of a hybrid solver that transitions from hyperbolic approximations of dispersive systems to the shallow water equations in different regimes. In particular, we deal with hyperbolic approximations of a dispersive depth-averaged Euler system [1] and the Serre-Green-Naghdi Equations [2]. The performance of this solver is benchmarked against standard experimental data and hypothetical tsunami scenarios.

References:

[1] Escalante, C., Dumbser, M., & Castro, M. J (2019). An efficient hyperbolic relaxation system for dispersive non-hydrostatic water waves and its solution with high order discontinuous Galerkin schemes. Journal of Computational Physics, 394, 385-416.

[2] Bassi, C., Bonaventura, L., Busto, S., & Dumbser, M. (2020). A hyperbolic reformulation of the Serre-Green-Naghdi model for general bottom topographies. Computers & Fluids, 212, 104716.

Solitary waves for dispersive equations with Coifman-Meyer nonlinearities

Wednesday
17:00

Johanna Ulvedal Marstrander
NTNU Norwegian University of Science and Technology

This talk concerns the existence of solitary-wave solutions to a class of nonlinear, dispersive evolution equations with Coifman-Meyer nonlinearities. Much effort has been put into answering whether there are solitary-wave solutions to a class of unidirectional, nonlinear wave equations, $\partial_t u + \partial_x(Mu + n(u)) = 0$, which arise in the study of water waves. Here, M is a possibly nonlocal, linear Fourier operator, whereas n is a local, nonlinear function. Inspired by several models where nonlinear frequency interaction appears, we extend the theory to allow for nonlocal nonlinearities in the form of bilinear Fourier multipliers or pseudo-products, $N(u, u)$. We establish the existence of smooth solitary waves to such equations when the linear multiplier is of positive and slightly higher order than the operator on the nonlinear term. The proof is based on a modified version of Weinstein's argument for L^2-constrained minimization using Lions' method of concentration-compactness.

Well-posedness of the Cummins equations

Wednesday
17:20

Martin Oen Paulsen
University of Bordeaux

The Cummins equation, introduced in the 1960s, serves as a linear model for the floating body problem in the irrotational setting. It has attracted a lot of attention in the engineering community since it allows for large-scale numerical simulations of floating structures with complex geometries. However, it is still an open question whether the system is well-posed.

Boundary conditions for the Boussinesq-Abbott model with varying bottom

Wednesday
17:40

David Lannes, Mathieu Rigal
Sorbonne University, France

In the littoral area, mechanisms behind the formation of extreme waves remain poorly understood despite their great socio-economic impact. In order to model these phenomena, it is especially important to take into account nonlinear and dispersive effects, which makes the Boussinesq-Abbott model a pertinent choice. However the presence of high order derivatives impedes the good handling of boundary conditions, which is crucial if one wishes to generate and evacuate waves from the computational domain. In order to raise this difficulty, an equivalent reformulation of this model has recently been proposed in the literature for the case of a flat bottom. This rewriting consists to get rid of the dispersive operator in exchange of a nonlocal flux and a dispersive boundary layer, and allows to efficiently prescribe the elevation of the free surface at the borders of the domain. The goal of this work is to extend this approach to the case of a varying bottom, while allowing to enforce more general boundary conditions. Once the nonlocal formulation of the model is established, numerical schemes of order 1 and 2 are proposed and validated through numerical experiments. The impact of different boundary conditions on the solutions is also investigated.

Wednesday
18:00

Global Bifurcation of Three-Dimensional Gravity-Capillary Waves on Beltrami Flows

Giang To
Lund University

Most of the current theory on three-dimensional steady water waves assumes irrotational flow. One exception is the construction of a family of small-amplitude doubly periodic gravity-capillary waves on Beltrami flows by Erik Wahlén, Lokharu and Svensson Seth from 2020. In the talk I will describe a global continuation of this family. One of the challenges is that the local family is constructed using a multiparameter bifurcation approach, whereas global bifurcation theory usually assumes a single bifurcation parameter. Our theory includes irrotational flow as a special case. This is joint work with Bastian Hilder and Erik Wahlén.

Thursday
9:00

Numerical solutions of the Hunter–Saxton equation

Katrin Grunert
Norwegian University of Science and Technology

Solutions of the Hunter–Saxton equation might enjoy wave breaking in finite time. This means that classical solutions in general do not exist globally, but only locally in time since their spatial derivative might become unbounded from below pointwise in finite time, while the solution itself remains bounded. In addition, energy concentrates on sets of measure zero when wave breaking occurs. As a consequence the prolongation of solutions beyond wave breaking is non-unique.

We will present a numerical method for α-dissipative solutions, i.e., solutions where the energy is manipulated at breaking time by taking out an α-part of the concentrated energy. This method combines a carefully chosen projection operator with a generalized method of characteristics and an iteration scheme, which enforces minimal time steps whenever wave breaking times cluster. Convergence is obtained for any admissible initial data, while a convergence rate is derived under some additional constraints on the initial data.

References:

T. Christiansen and K. Grunert, A numerical view on α-dissipative solutions of the Hunter–Saxton equation, ESAIM Math. Model. Numer. Anal. 59 579–612 (2025).

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Recent results on traveling water waves

Thursday
9:40

Jörg Weber
University of Vienna

While the research on water waves modeled by Euler's equations has a long history, mainly in the last two decades traveling periodic rotational waves have been constructed rigorously by means of bifurcation theorems. After introducing the problem and putting it into context, I will present a new reformulation in two dimensions in the pure-gravity case, where the problem is equivalently cast into the form "identity plus compact," which is amenable to Rabinowitz's global bifurcation theorem. The main advantages (and the novelty) of this new reformulation are that no simplifying restrictions on the geometry of the surface profile and no simplifying assumptions on the vorticity distribution (and thus no assumptions regarding the absence of stagnation points or critical layers) have to be made. Within the scope of this new formulation, global families of solutions, bifurcating from laminar flows with a flat surface, are constructed. Moreover, I will discuss the possible alternatives for the global set of solutions, as well as their nodal properties. This is joint work with Erik Wahlén.

Numerical integration of the nonlinear Schrödinger equation with absorbing boundary conditions.

Thursday
10:50

Nuria Reguera
University of Burgos, Spain

When we wish to solve numerically a differential problem defined on an infinite domain, it is necessary to consider a finite subdomain and to use artificial boundary conditions. The optimal result occurs when the solution of the constructed initial boundary value problem coincides with the constraint of the solution of the original problem to the chosen finite subdomain. The boundary conditions that provide this result are called transparent boundary conditions (TBCs). However, often TBCs are nonlocal so they have a high computational cost and their practical interest is limited. In these cases, the so-called absorbing boundary conditions (ABCs) are preferred, which are local, are constructed as an approximation of the transparent ones and allow small reflections of the numerical solution when it reaches the boundary.

In the case of the Schrödinger equation, obtaining ABCs has been extensively studied for the linear case, while in the case of the nonlinear Schrödinger equation the literature is much more sparse.

In this talk we will focus on the numerical integration of the nonlinear Schrödinger equation. We will consider a splitting method for integration in time which will allow us to approximate the nonlinear Schrödinger equation using ABCs only for the linear case.

This is a joint work with Rolando Muñoz (Univ. Autónoma de Aguascalientes, México).

Thursday
11:30

Numerical solution of nonlinear dispersive wave problems with symmetric partitioned linear multistep methods

B. Cano and A. Durán

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In this talk the use of partitioned linear multistep methods (PLMM) as time integrator for the numerical approximation of PDEs is introduced. First, we study the time behaviour of the global error of PLMM when integrating partitioned odes, following [1]. We then consider the periodic initial-value problem of the Bona-Smith systems for surface wave propagation as case study. From the spatial discretization with spectral methods, the time integration of the semidiscrete system with symmetric PLMMs, when approximating localized solutions is analyzed and illustrated with numerical experiments.

References:

[1] B. Cano, A. Durán, M. Rodríguez, Long-term behaviour of symmetric partitioned linear multistep methods I. Global error and conservation of invariants, in Press.

[2] B. Cano, A. Durán, Long-term behaviour of symmetric partitioned linear multistep methods II. Analysis for some nonlinear dispersive wave problems, in Press.

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